





**Solution:** Because of the nature of the labels (just naturals, no tuples), we assume that the automaton was constructed from a formula that solely consisted of one atomic formula. We notice  $q_t \notin \delta(Q, \Sigma)$ , thus we can't assume an equation, instead we have to aim for an inequation.

We try to find upper and lower bounds to the coefficients by reversing the computation performed to find the transition function.

$$j = \left\lfloor \frac{1}{2}(k - a\zeta) \right\rfloor$$

$$\frac{1}{2}(k - a\zeta) \in [j, j + 1]$$

$$k - a\zeta \in [2j, 2(j + 1)[$$

$$-a\zeta \in [2j - k, 2(j + 1) - k[$$

$$a\zeta \in ]k - 2(j + 1), k - 2j]$$

$$a\zeta \in [k - 2(j + 1) + 1, k - 2j]$$

$$a\zeta \in [k - 2j - 1, k - 2j]$$

Using these bounds we can now define a Presburger formula that specifies a.

$$\bigwedge_{\substack{\zeta \in \{0,1\}^n \\ q \in Q}} q - 2\delta(q, \zeta) - 1 \leq a\zeta \wedge a\zeta \leq q - 2\delta(q, \zeta)$$

For this example we yield:

$$\begin{array}{lcl} -2 \leq a_1 \wedge a_1 \leq -1 & \wedge & -1 \leq a_1 \wedge a_1 \leq 0 \\ \wedge & & \wedge \\ 1 \leq a_2 \wedge a_2 \leq 2 & \wedge & 2 \leq a_2 \wedge a_2 \leq 3 \\ \wedge & & \wedge \\ 0 \leq a_1 + a_2 \wedge a_1 + a_2 \leq 1 & \wedge & 1 \leq a_1 + a_2 \wedge a_1 + a_2 \leq 2 \end{array}$$

Simplification gets us the following:

$$a_1 = -1 \wedge a_2 = 2$$

From  $q_0$  we get b and can finally piece  $\phi$  together:

$$\varphi = -1x_1 + 2x_2 \leq -3$$

$\{(x_1, 3), (x_2, 0)\}$  is one possible satisfying assignment.

### Exercise 4.3 Construct

Construct automata for the formulas and provide a satisfying assignment for the free variables in decimal notation. The last one is a bonus task and will not be graded, there won't be a solution either.

$$\varphi_1 := x_1 - 2x_2 \leq 3, \varphi_2 := -2x_1 - x_2 \leq -4, \varphi := \varphi_1 \vee \varphi_2$$

Solution:

