

## Algorithms for the Undecided – Homework 6

Discussed on Friday, 8<sup>th</sup> July, 2016.

### Exercise 6.1     **Sturm Sequence**

Build the Sturm sequence of the following polynomial:

$$P = X^4 - 4X^3 - X^2 + 16X - 12$$

Count the number of roots of  $P$  in  $\mathbb{R}$  with the help of the Sturm sequence.

*Hint:* All polynomials in the Sturm sequence can be multiplied with positive constants without changing the outcome of the algorithm and for getting nicer looking integer coefficients in the polynomials.

### Exercise 6.2     **Sturm Sequence**

Consider the following equation and inequality:

$$\begin{aligned}x^3 + 3x^2 - 4x - 12 &= 0 \\ -x &> 0\end{aligned}$$

Use the method from the seminar with Sturm sequences to check if there exists a real solution to this system.

*Hint:* You only need to show the existence of a solution.

### Exercise 6.3     **System of inequalities**

Please ignore this exercise.

~~Use the method described on the slides to decide if there exists a real solution to the following system of inequalities:~~

$$\begin{aligned}x^2 + 3x - 4 &> 0 \\ x &> 0\end{aligned}$$

### Exercise 6.4     **Quantifier elimination**

Use quantifier elimination to transform the following formula to a system depending only on  $a, b$  and  $c$ :

$$\exists x. ax^2 + bx + c = 0$$