Interactive Software Verification

Spring Term 2013

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16.4.2013

Goals of the Lecture

- Formulate precise statements and arguments about programs
- Develop a language for talking about correctness
- Understand fundamental concepts of software verification
- Explain your thought to a computer / program
  - “To understand this properly, we implement it”

  $\Rightarrow$ “To understand this precsiely, we explain it to Isabelle”
- Online demo: `double`/`even`; `add`

Motivation for Verification

- Debugging & maintenance are time-consuming & expensive
- Embedded systems
  - Control safety-critical devices (air bag, planes)
  - Replacement only by physical updates
- Foundational routines ⇨ multiplication of errors
  - Libraries & data structures
  - Infracstructure: memory manager, garbage collectors
- Basic software
  - Operating systems
  - Hypervisors
  - Compilers

Examples of Interactive Verification

- C compiler [4, 15]
- L4 micro-kernel [12, 11, 21]
- Safetly of robot movements [16]
- JavaCard API [8]
- Garbage collectors [17]
- Security protocols [20]
- Algorithmic questions [6, 18]

Examples of Automatic Verification

- Libraries of data structures [23]
- Garbage collectors [10, 14]
- MS Hypervisor (VerifsoftXT project) [1]
- Sanity checks for Java programs [9]
- Concurrent programs [13]
- Algorithms on lists, trees, . . . [7, 22, 3]

Why get involved at all?

- "Automatic" methods still require algorithmic insights
  - Annotating programs with assertions about state
  - Maintaining ghost state with logical/abstract state
  - Stating lemmas / auxiliary assertions
- The proof process needs to be understood in some detail
  - Control the proof search of SMT solvers [19]
  - Provability of given goals often not immediately clear [5]
  - Beginners need support by experts [2]

$\Rightarrow$ This lecture: explore concepts by interactive proof

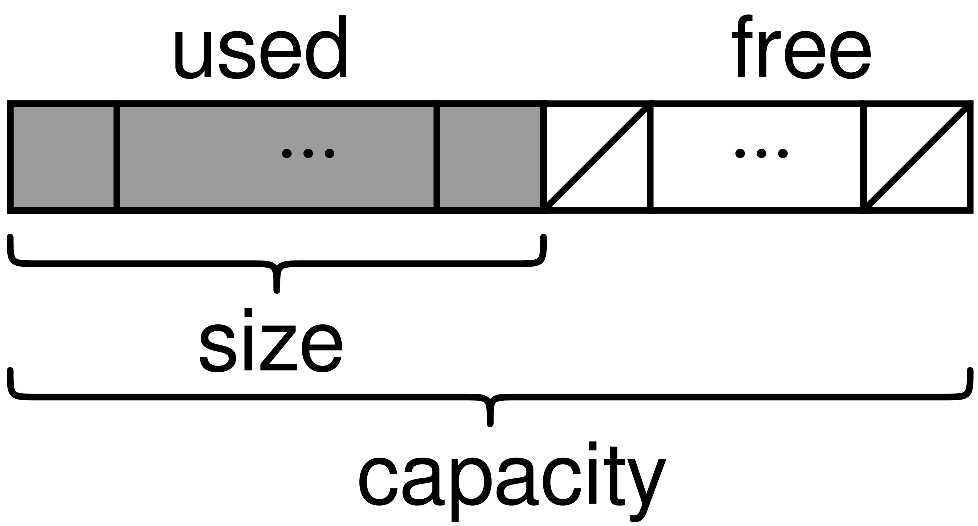
How “Interactive” are Interactive Provers?

- Interative prover: Isabelle
- User
  - Defines notions / terms of interest
  - States theorems about definitions
  - Guides Isabelle towards finding proofs
- Extensive automatic proof support
  - Term rewriting (proofs about equality)
  - Resolution in first order predicate logic (FOL)
  - Tableau prover for higher order logic (HOL)
- Proof automation will be discussed as necessary

What does “correct” mean anyway?

```java
public class ArrayList<E> {
  private Object[] data;
  private int size;
  public void add(E e) {
   ensureCapacity(size + 1);
   data[size++] = e;
  }
  public void ensureCapacity(int minCapacity) {
    int oldCap = data.length;
    if (minCapacity > oldCap) {
      Object oldData[] = data;
      int newCap = (oldCap * 3)/2 + 1;
      if (newCap < minCapacity)
        newCap = minCapacity;
      data = Arrays.copyOf(data, newCap);
    }
  }
}
```

ArrayList – Inner Structure



- The first `size` slots contain object references (or null)
- The remaining slots in `elementData` are `null` (GC!)

Arguments about Correctness

Question: how can we convince a very sceptical person that our program is correct? About what do we argue?

- Content of object fields at specific points of program
- Result of method calls
- Relation between parameters and return value of methods
- Information gained by case distinctions
- Result / effect of assignment
- Consistency of objects

Application to `ArrayList`

```
public class ArrayList<E> {
  private Object[] elementData;
  private int size;
  public boolean add(E e) {
   ensureCapacity(size + 1);
   // Can insert the new element since enough space is 'free',
   // i.e. the element 'size' is not occupied since
   // elementData.length > size, i.e. there will not be
   // ArrayOutOfBoundsException
   // Reason: ensureCap guarantees .length >= size + 1
   elementData[size++] = e;
   return true;
  }
  public void ensureCapacity(int minCapacity) {
    int oldCapacity = elementData.length;
    if (minCapacity > oldCapacity) {
      // minCapacity > elementData.length
      Object oldData[] = elementData;
      int newCapacity = (oldCapacity * 3)/2 + 1;
      if (newCapacity < minCapacity)
        newCapacity = minCapacity;
      elementData = Arrays.copyOf(elementData, newCapacity);
      // elementData.length = newCapacity && newCapacity >= minCapacity.
    } else {
      // elementData.length >= minCapacity
    }
    // at this point *in any case* elementData.length >= minCapacity;
  }
}
```

(created interactively)

The Big Picture

Correctness of Programs
Solve verification conditions Arguments about application domain
Hoare Logic
Verification rules for language constructs Generator for verification conditions
Semantics
Define meaning of programs Describe behaviour of programs

The Case for Java

Correctness of Programs
Solve Proof Obligations
Hoare Logic
Pre-/Post-Conditions for if, while, try, catch Pre-/Post-Conditions in connection with inheritance Disjointness of objects and fields
Semantics
Objects, References, Arrays Expressions, Statements, Exceptions Dynamic Dispatch

The Case for C

Correctness of Programs
Solve proof obligations Aliasing of pointers
Hoare Logic
Expressions, . . . as in Java Untyped memory access / casts
Semantics
Byte-addressed memory, bit manipulations Allocation of memory regions Pointer arithmetic and -casts Pointers to local variables / struct fields

Demo: Multiplication by Addition

```
verify-statement mult-by-add
vars: {*
 int i;
 int j;
 int r;
*}
pre: "i = I ∧ j = J ∧ i ≥ 0 ∧ j ≥ 0"
post: " r = I * J"
{*
r = 0;
/*@ r = (I - i) * j ∧ j = J ∧ i ≥ 0*/
while (i > 0) {
 r = r + j;
 i = i - 1;
}
*}
```

The Plan for this Lecture

- Start small, then advance to the more complex
  - Look at simple (functional) languages
  - Learn interactive proving
  - Define imperative language with variables, heap, etc.
  - Prove properties about heap manipulating programs
- Demonstrate / experience the concepts in the small
  - Lecture presents concepts
  - Reduction to main aspects
  - Implementation & use of core aspects in Isabelle

Start with Isabelle's Language

Correctness of Programs
Proofs about defined functions
Verification Logic
Express statements directly as theorems
Semantics
Recursive functions ($\approx$ ML, Haskell, Scheme) Standard data types (natural numbers, lists, . . .)

Example: Binary Trees

- Definition: a tree is either
  - A leaf *or*
  - An inner node with two sub-trees

```
datatype bt =
  Leaf
| Node bt int bt
```

- Concept: the elements of a tree

```
fun in-order :: "bt ⇒ int list"
where
 "in-order Leaf = []"
| "in-order (Node l v r) = in-order l @ [ v ] @ in-order r"
definition
 "bt-elems t ≡ set (in-order t)"
definition
 "bt-contains t x ≡ x ∈ bt-elems t"
```

Algorithm: Search in Tree

- Naive implementation

  fun bt-search-full :: "bt ⇒ int ⇒ bool"
  where
  "bt-search-full Leaf x = False"
  | "bt-search-full (Node l v r) x =
  (v = x ∨ bt-search-full l x ∨ bt-search-full r x)"

- Observations
  - We can program in Isabelle
  - Semantics: just as in functional languages
  - We can use mathematical concepts, like "∨", "∃"

Executing the Code

export-code bt-search-full in OCaml
module-name BTSearchFull
file "code/btsearchfull.ML"

Generates an executable (OCaml-) program:

```
type bt = Leaf | Node of bt * int * bt;;

let rec bt_search_full
xa0 x = match xa0, x with Leaf, x -> false
| Node (l, v, r), x ->
  eq_int v x || (bt_search_full l x || bt_search_full r x);;
```

One can write entire C compilers in this way [4, 15]

Correcteness of Isabelle Functions

- Why is bt-search-full "correct"?
  - It yields "`True`" iff `x` is element of the tree

$\Rightarrow$ Given by bt-contains

- Formulate as lemma ⇨ proof obligation (goals)

  lemma bt-search-full-correct:
  "bt-search-full t x $\longleftrightarrow$ bt-contains t x"

- Proof

  Induction on the tree structure
  apply (induct t)
  Use the definitions
  apply (unfold bt-contains-def bt-elems-def)
  Solve remainder automatically
  apply auto
  done

Binary Search in Trees

- A tree is sorted if its elements by in-order traversal are sorted

  definition
  "bt-sorted t ≡ sorted (in-order t)"

- Now we can search by divide-and-conquer

  fun bt-search-split :: "bt ⇒ int ⇒ bool"
  where
  "bt-search-split Leaf x = False"
  | "bt-search-split (Node l v r) x =
  (v = x ∨ (**if** x < v
  then bt-search-split l x
  else bt-search-split r x))"

Correctness of Binary Search

- For sorted trees the new function is correct

  lemma bt-search-split-correct:
  $''\text{bt-sorted}\ t \Longrightarrow \text{bt-search-split}\ t\ x \longleftrightarrow \text{bt-contains}\ t\ x''$

  - Correctness condition as before
  - Now requires assumption / pre-condition on sortedness
- Proof
  - Structural induction on t
  - Auxiliary facts (immediate by definitions)

    lemma bt-sorted-branchesD:
    $''\text{bt-sorted}\ (\text{Node}\ l\ v\ r) \Longrightarrow \text{bt-sorted}\ l \wedge \text{bt-sorted}\ r''$
    lemma bt-sorted-elemsD:
    $''\text{bt-sorted}\ (\text{Node}\ l\ v\ r) \Longrightarrow$
    $(\forall x \in \text{bt-elems}\ l.\ x \leq v) \wedge (\forall x \in \text{bt-elems}\ r.\ v \leq x)''$

Data structures & Invariants

- Consider insertion into a binary tree

```
fun bt-insert :: "[int, bt] ⇒ bt"
where
 "bt-insert x Leaf = Node Leaf x Leaf"
| "bt-insert x (Node l v r) =
        (if x = v
         then (Node l v r)
         else if x < v
              then Node (bt-insert x l) v r
              else Node l v (bt-insert x r))"
```

- Descend to the "correct" place recursively
- Then add the element

Correctness of bt-insert

- Invariant: sorted trees remain sorted after insertion

  lemma bt-insert-is-sorted:
  "bt-sorted t $\Longrightarrow$ bt-sorted (bt-insert x t)"

- And the inserted element is actually contained in the tree

  lemma bt-insert-result-set:
  "bt-elems (bt-insert x t) $=$ $\{x\}$ $\cup$ bt-elems t"

- Proofs
  - By induction on t
  - Auxiliary facts on sorted trees as before

Verwendung der Korrektheitsaussagen

- Now: serveral function applications in a row
- What can we know about the result of:

lemma
"bt-sorted t $\Longrightarrow$
let t' = bt-insert x t in
let t'' = bt-insert y t' in
bt-search-split t'' x = True"

- Need auxiliary facts about rows of function applications

lemma bt-contains-bt-insert:
"bt-contains (bt-insert x t) y = (y = x $\vee$ bt-contains t y)"
apply (simp add: bt-contains-def)
apply (simp add: bt-insert-result-set)
done

- Then we can prove the above lemma

by (simp add: Let-def bt-insert-is-sorted
bt-search-split-correct bt-contains-bt-insert)

Conclusion: Isabelle as a Proof Assistant

- Define concepts & functions
- Formulate statements about these
- Proof these statements
  - Obvious parts done automatically
  - Can structure proof along informal arguments

  $\Rightarrow$ Proof $\approx$ very precise version of explanation
- The „shoulders of a giant“
  - Extensive libraries of standard concepts available
  - Frequent goals about concepts automated

Content of the Lecture

- Interactive proofs with Isabelle
- Semantics of programming languages
- Hoare-Logics for imperative languages
- Techniques & heuristics for verification
- Heap manipulating programs
  - Burstall's memory model
  - Separation Logic

What You can Learn

1. How to argue about the correctness of (imperative) software
   - Datastructures, state, memory modification
   - Making informal arguments precise
2. Concepts underlying formal correctness arguments
   - Semantics (meaning) of programming languages
   - Hoare logic & verification conditions
   - Heap models
3. Lab excercises: small exemplary proofs
   - Get a detailed grasp of concepts
   - Introduction to Isabelle

Organisation

- Mode: teams (of 2) work together during lab
- If need be: prove remainder at home
- Goal: define concepts & find example proofs along lecture
- We will now find a common time for the exercises
- Examns: oral or written, depending on number of participants

Final remarks

- I hope I have been able to . . .
  - inform you
  - get you interested
  - motivated you
  - challenged you
- . . . such that we meet again next week!

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