

### Exercise 1 (Confluence & Commutation)

Show: If  $\rightarrow_1$  and  $\rightarrow_2$  are confluent, and if  $\rightarrow_1^*$  and  $\rightarrow_2^*$  commute, then  $\rightarrow_{12} := \rightarrow_1 \cup \rightarrow_2$  is also confluent.

### Exercise 2 (Local Commutation)

Show: If

$$t_2 \leftarrow s \rightarrow_1 t_1 \implies \exists u. t_2 \rightarrow_1^= u \xrightarrow{*}_2 \leftarrow t_1,$$

then  $\rightarrow_1^*$  and  $\rightarrow_2^*$  commute.

Here  $\rightarrow^=$  denotes the reflexive closure of  $\rightarrow$ , i.e.:

$$\rightarrow^= := \rightarrow \cup \rightarrow^0$$

### Exercise 3 (Strong Confluence)

A relation  $\rightarrow$  is said to be *strongly confluent* iff:

$$t_2 \leftarrow s \rightarrow t_1 \implies \exists u. t_2 \rightarrow^* u \xrightarrow{=}_1 \leftarrow t_1$$

Show that every *strongly confluent* relation is also *confluent*.

### Homework 4 (Semi-Confluence)

A relation  $\rightarrow$  is said to be *semi-confluent* iff:

$$t_2 \xrightarrow{*} \leftarrow s \rightarrow t_1 \implies \exists u. t_2 \rightarrow^* u \xrightarrow{*} \leftarrow t_1$$

Show that  $\rightarrow$  is *semi-confluent* if and only if it is *confluent*.

### Homework 5 (Diamond Property & Normal Forms)

Show that if  $\rightarrow$  has the diamond property, every element is either in normal form or has no normal form.

### Homework 6 (Weak Diamond Property)

Assume that  $\rightarrow$  has the following weaker diamond property:

$$t_2 \leftarrow s \rightarrow t_1 \wedge t_1 \neq t_2 \implies \exists u. t_2 \rightarrow u \leftarrow t_1.$$

- Is it still the case that every element is either in normal form or has no normal form?
- Show that if  $t$  has a normal form, then all its reductions to its normal form have the same length.