

### Exercise 1 (Better Translation Algorithm)

Give a variant of the translation algorithm that produces shorter terms. More specifically, define a variant of  $\lambda^*x.t$  that analyzes more precisely where  $x$  actually appears in  $t$ .

### Exercise 2 (Confluence of $\rightarrow$ )

Introduce a parallel reduction relation  $>$  for combinatory logic and show that it fulfills the diamond property.

### Homework 3 (More Combinators)

Find combinators  $O$  and  $W$  such that:

$$\begin{aligned} O &\rightarrow^+ O \\ WXY &\rightarrow^* XYY \end{aligned}$$

### Homework 4 (Mocking Birds)

Consider a combinatory logic that only provides the basic combinators  $B$  and  $M$  (the “mocking bird”) where:

$$\begin{aligned} BXYZ &\rightarrow^* X(YZ) \\ MX &\rightarrow^* XX \end{aligned}$$

Prove the following properties of this logic:

- For every combinator  $X$ , there is a combinator  $Y$  such that  $Y \rightarrow^* XY$ .
- For all combinators  $U$  and  $W$ , there exist combinators  $X$  and  $Y$  such that  $Y \rightarrow^* UX$  and  $X \rightarrow^* WY$ .

### Homework 5 (A Different Basis)

Show that  $B$ ,  $C$ ,  $K$  and  $W$  are also sufficient to represent all  $\lambda$ -terms, where

$$CXYZ \rightarrow XZY$$

## Homework 6 (Correctness of the Translation Algorithm)

Show that the translation algorithm given in the tutorial is correct. That is, show that it fulfills the following property:

$$(\lambda^*x. X) Y \rightarrow^* X[Y/x]$$