

Exercise 1 (Combinators for Shorter Proofs)

We have introduced the combinators **B** and **C** to obtain shorter CL-translations of λ -terms. What axioms do these correspond to in the Hilbert system?

Exercise 2 (Proof Translation)

Recall our proof of

$$((((P \rightarrow Q) \rightarrow P) \rightarrow P) \rightarrow Q) \rightarrow Q$$

in intuitionistic logic. Translate the proof to the Hilbert system using the axioms from the last exercise (in addition to the ones known from the lecture).

Exercise 3 (Proof in the Hilbert System)

Prove in the Hilbert System:

$$(P \rightarrow Q) \rightarrow (R \rightarrow P) \rightarrow R \rightarrow Q$$

Exercise 4 (Extending the System)

Suppose we want to extend the Hilbert system with \wedge and \vee . What would the corresponding new axioms, combinators and proof translation rules look like?