

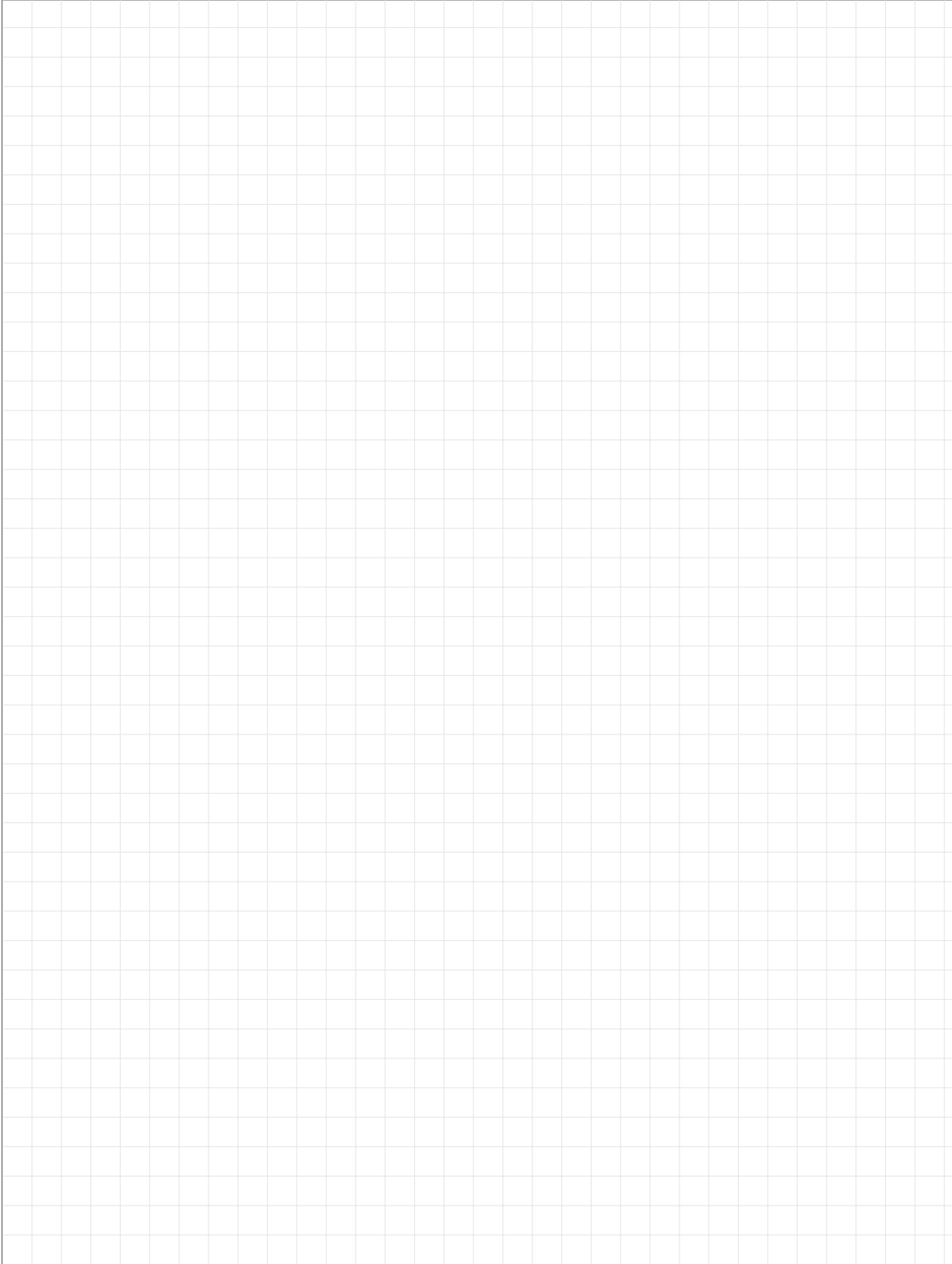
Exercise 1 (Progress Property)

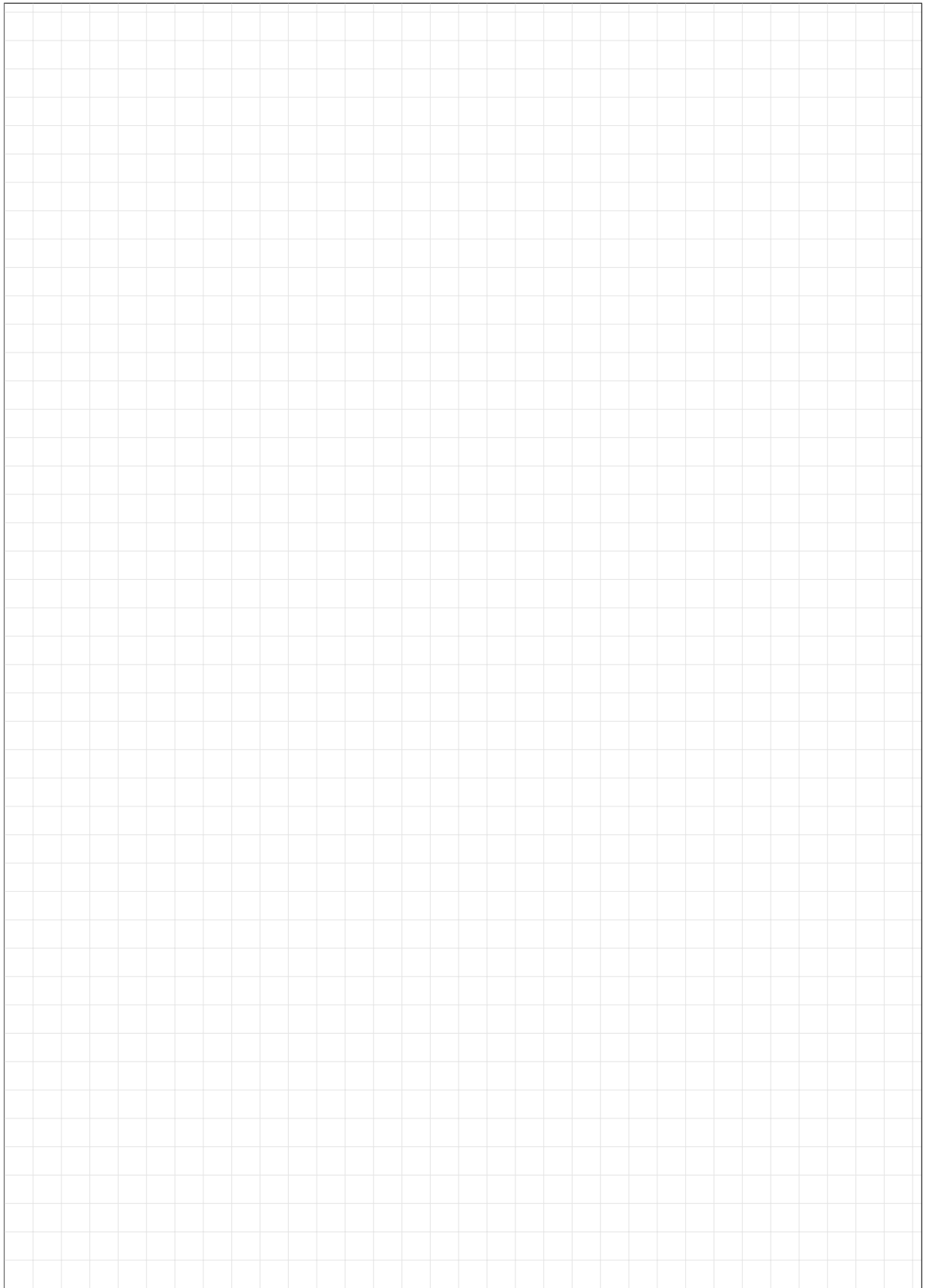
Let t be a closed and well-typed term, i.e. $[] \vdash t : \tau$ for some τ . Show that t is either a value or there is a t' such that $t \rightarrow_{cbv} t'$.



Exercise 2 (Normal Form)

Show that every type-correct λ^{\rightarrow} -term has a β -normal form.





Homework 3 (Typing)

a) Prove:

$$\boxed{\ } \vdash (\lambda x: \tau_2 \rightarrow \tau_3. \lambda y: \tau_1 \rightarrow \tau_2. \lambda z: \tau_1. x (y z)) : (\tau_2 \rightarrow \tau_3) \rightarrow (\tau_1 \rightarrow \tau_2) \rightarrow \tau_1 \rightarrow \tau_3$$

b) Give suitable solutions for $?\tau_1$, $? \tau_2$, $? \tau_3$ and $? \tau_4$ and prove that the term is type-correct given your solution.

$$\boxed{\ } \vdash \lambda x: ?\tau_1. \lambda y: ?\tau_2. \lambda z: ?\tau_3. x y (y z) : ?\tau_4$$

Homework 4 (β -reduction preserves types)

A type system has the *subject reduction property* if evaluating an expression preserves its type. Prove that the simply typed λ -calculus (λ^{\rightarrow}) has the subject reduction property:

$$\Gamma \vdash t: \tau \wedge t \rightarrow_{\beta} t' \implies \Gamma \vdash t': \tau$$

Hints: Use induction over the inductive definition of \rightarrow_{β} (Def. 1.2.2). State your inductive hypotheses precisely – it may help to introduce a binary predicate $P(t, t')$ to express the property you are proving by induction. Also note that the proof will require *rule inversion*: Given $\Gamma \vdash t: \tau$, the shape of t (variable, application, or λ -abstraction) may determine which typing rule must have been used to derive the typing judgment.

Within your proof, you are free to use the following lemma about substitution:

$$\Gamma \vdash u: \tau_0 \wedge \Gamma[x: \tau_0] \vdash t: \tau \implies \Gamma \vdash t[u/x]: \tau \tag{1}$$

Homework 5 (Implementation of multiset-ordering and reduction)

Implement the multiset ordering and the reduction strategy from the second tutorial exercise in your favorite programming language.