

Exercise 1 (Intuitionistic Proof Search)

The algorithm in Theorem 4.1.4 can be streamlined as follows:

- a) When trying to prove $\Gamma \vdash A \rightarrow B$, it suffices to try (\rightarrow Intro). Explain why.
- b) The attempt to prove $\Gamma \vdash A$ by assumption can be dropped: it is subsumed by the alternative using Lemma 4.1.2. However, the proof obtained can be different. Explain the difference and why the outright proof by assumption is subsumed.
- c) How would the Haskell code from the last tutorial need to be adopted to account for these improvements?

Exercise 2 (Intuitionistic Proofs)

Prove the following propositions in intuitionistic logic:

- a) $(A \rightarrow A) \vee B$
- b) $A \rightarrow (B \rightarrow A \wedge B)$
- c) $(A \rightarrow C) \rightarrow ((B \rightarrow C) \rightarrow (A \vee B \rightarrow C))$

Homework 3 (Weak Normalization with Pairs)

We previously proved (sheet eight, ex. two) that every type-correct λ^{\rightarrow} -term has a β -normal form. Adapt the proof to accommodate for the extension of the simply typed lambda calculus with pairs.

Homework 4 (From Proof Terms to Propositions)

Consider the following proof term:

$$\lambda q. \lambda p. \text{case } \pi_1 p \text{ of } \text{Inl } a \Rightarrow \text{Inl } (\pi_1 q, (a, \pi_2 p)) \mid \text{Inr } b \Rightarrow \text{Inr } (\pi_2 q, b)$$

- a) Exhibit the proposition that is proved by this term.
- b) Give the corresponding proof tree.

Homework 5 (Intuitionistic Proofs)

Prove the following propositions in pure logic, without lambda-terms, and write down the λ -term corresponding to each proof:

- a) $\neg(A \vee B) \rightarrow \neg A \wedge \neg B$
- b) $\neg A \wedge \neg B \rightarrow \neg(A \vee B)$