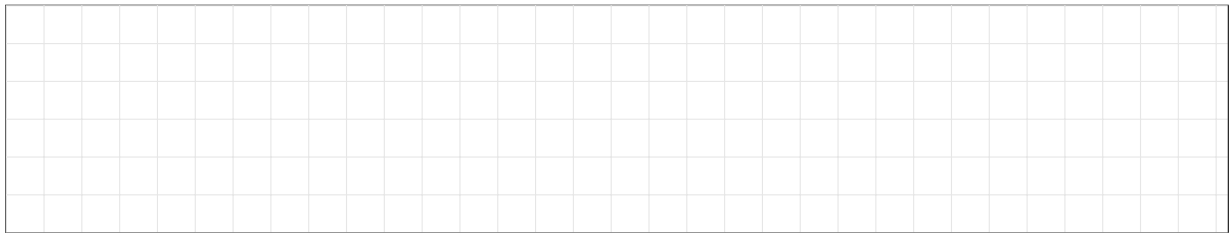


### Exercise 1 ( $\lambda$ -Terms)

Rewrite the following terms such that they are completely parenthesized and conform to the grammar for the  $\lambda$ -calculus given in the lecture (without any shortcut notations).

a)  $u x (y z) (\lambda v. v y)$

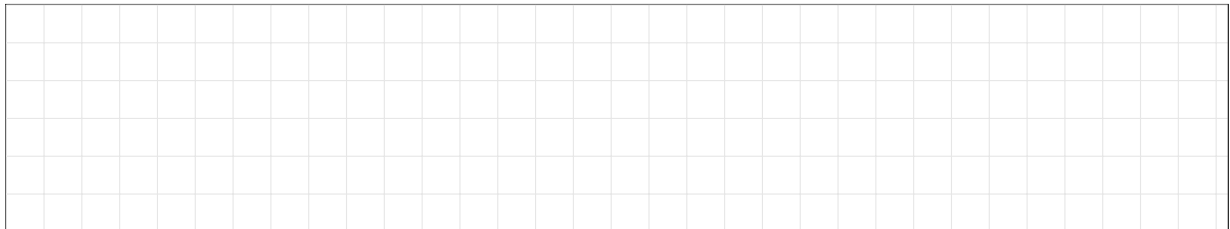
b)  $(\lambda x y z. x z (y z)) u v w$



Rewrite the following terms such that there are as few parentheses as possible, and apply all shortcut notation from the lecture:

c)  $((u (\lambda x. (v (w x)))) x)$

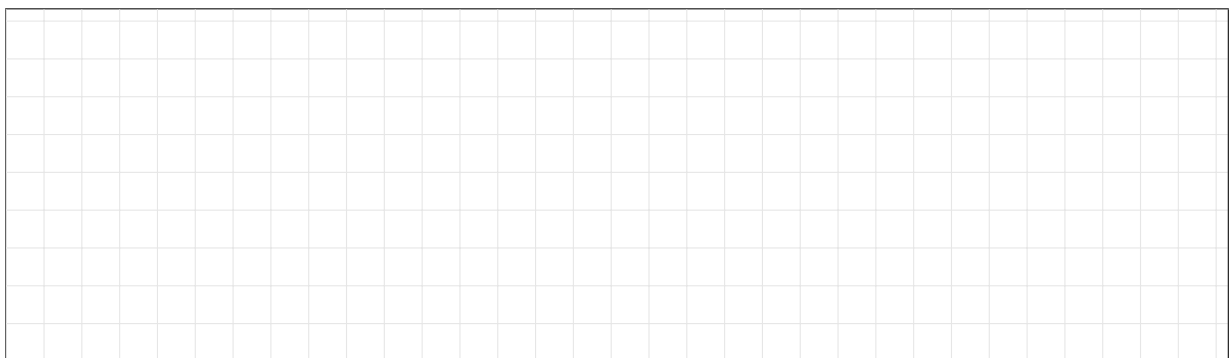
d)  $((((w (\lambda x. (\lambda y. (\lambda z. ((x z) (y z)))))) u) v)$

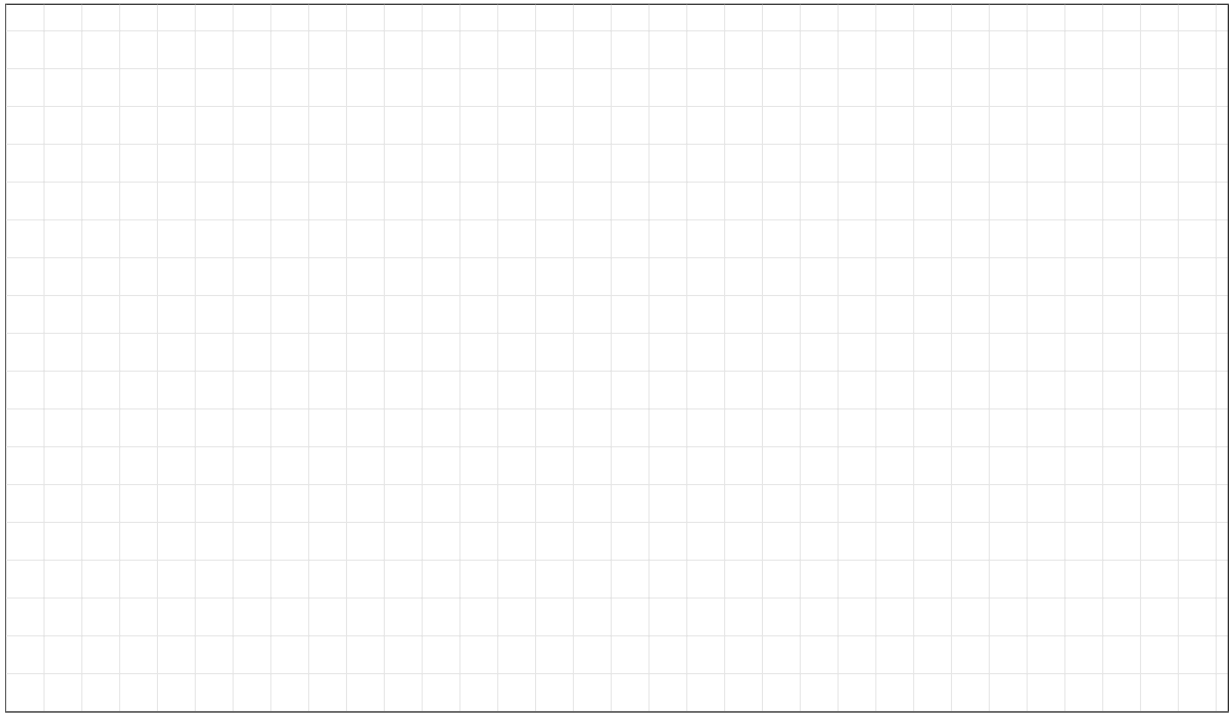


Evaluate the following substitutions:

e)  $(\lambda y. x (\lambda x. x)) [(\lambda y. x y)/x]$

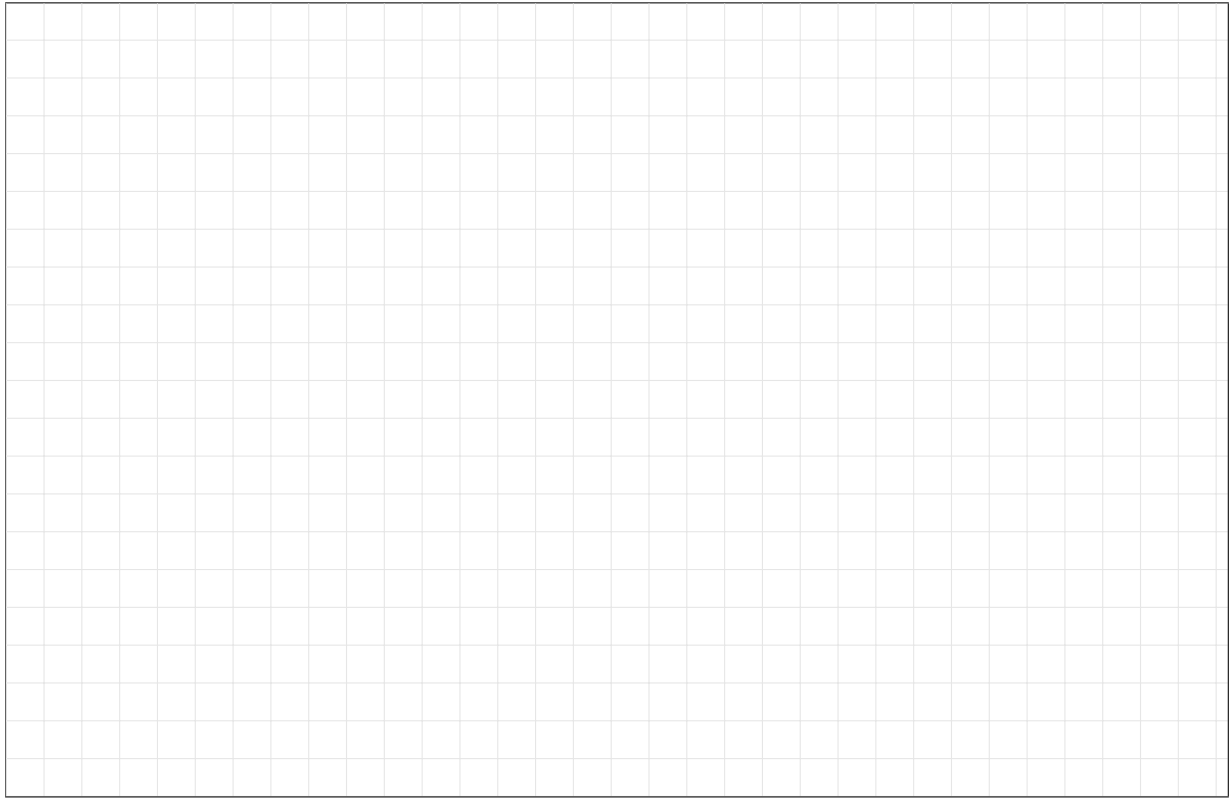
f)  $(y (\lambda v. x v)) [(\lambda y. v y)/x]$





**Exercise 2 ( $\lambda$ -Terms as Trees)**

Rewrite the  $\lambda$ -terms resulting from exercises 1c) and 1d) to their corresponding representation as a tree.

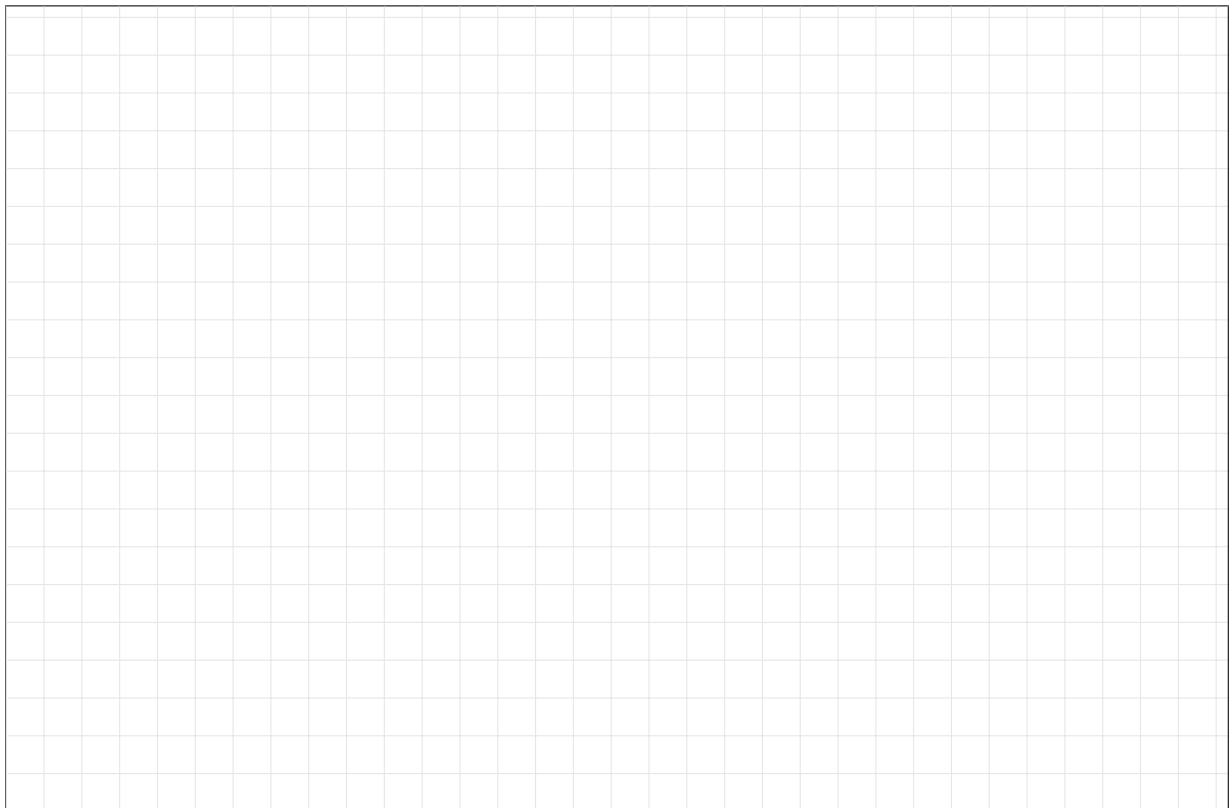


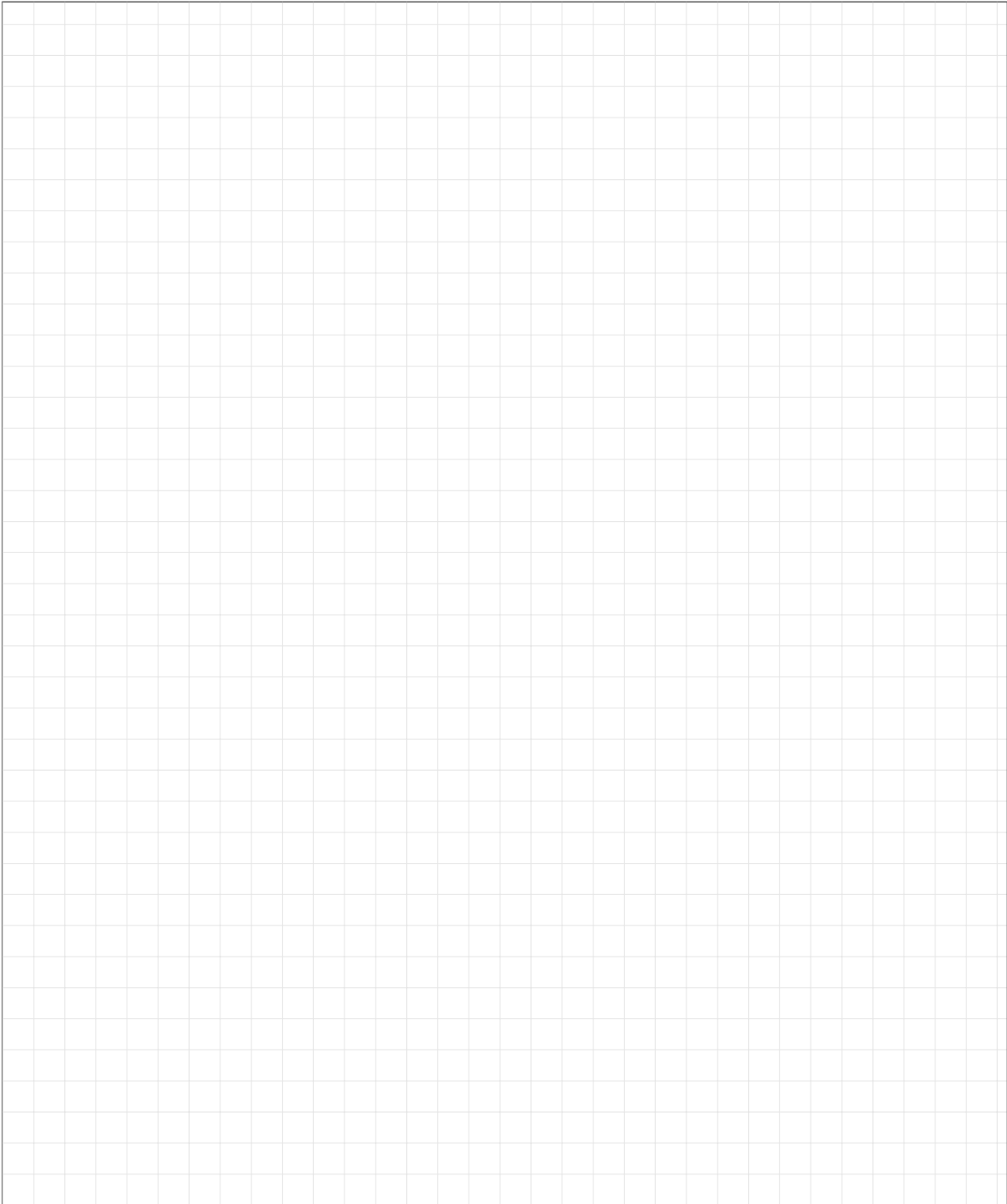


### Exercise 3 (Formalisations with $\lambda$ -Terms)

Express the following propositions as  $\lambda$ -terms. Use the constant  $D$  as a derivative operator.

- a) The derivative of  $x^2$  is  $2x$ .
- b) The derivative of  $x^2$  at 3 is 6.
- c) Let  $f$  be a function, and let  $g$  be defined as  $g(x) := f(x^2)$ . The derivative of  $g$  at  $x$  is different from the derivative of  $f$  at  $x^2$ .
- d) Formulate the proposition c) without using the auxiliary function symbol  $g$ .





#### Homework 4 (Interpreting $\lambda$ -Terms)

Give a *compact* natural-language description of the computational effect of the following  $\lambda$ -terms.

- a)  $\lambda x. x$
- b)  $\lambda x y. x$
- c)  $\lambda x y z. x z y$
- d)  $\lambda x y. x (x y)$
- e)  $\lambda x y z. x (y z)$

#### Homework 5 (Free and Bound Variables)

Mark the free variables in the following examples. Graphically indicate (by drawing arrows) the binding  $\lambda$  for each bound variable.

- a)  $\lambda x y z. (\lambda x y. z x) y (x z)$
- b)  $\lambda x. \lambda y. (\lambda y. z (\lambda z. y x)) (\lambda x z. x y z) y x$

#### Homework 6 (Substitutions)

Evaluate the following substitutions:

- a)  $((\lambda x. f x) (\lambda f. f x)) [g x / f]$
- b)  $(\lambda f. \lambda y. f x y) [f y / x]$

#### Homework 7 (Properties of Substitution)

Evaluate the following substitutions:

- a) Give a counterexample for

$$s[t/x][u/y] = s[u/y][t/x].$$

- b) Under which conditions does

$$s[t/x][u/y] = s[t[u/y]/x]$$

hold?