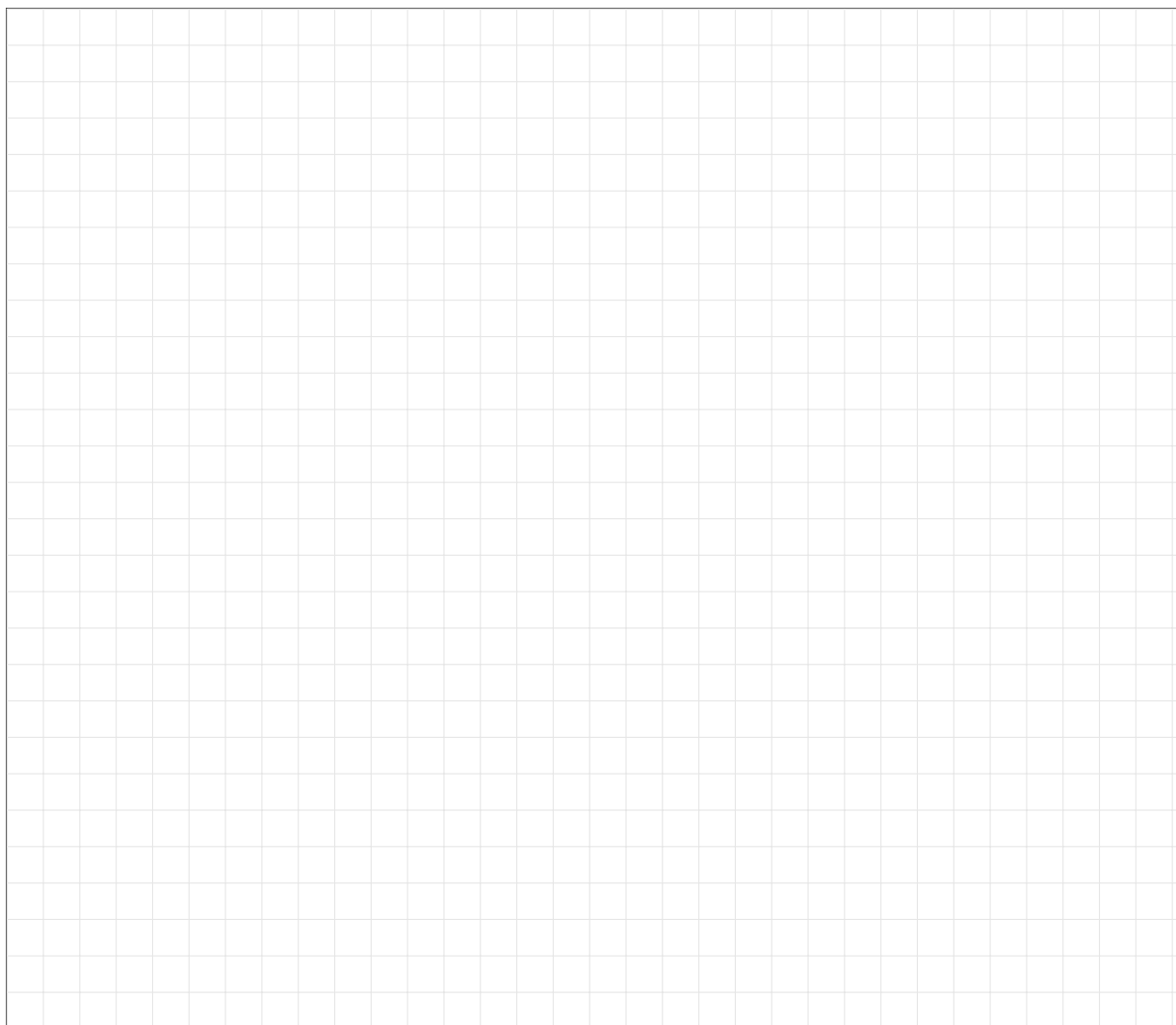


### Exercise 1 (Intuitionistic Proof Search)

The algorithm in Theorem 4.1.4 can be streamlined as follows:

- a) When trying to prove  $\Gamma \vdash A \rightarrow B$ , it suffices to try ( $\rightarrow$ Intro). Explain why.
- b) The attempt to prove  $\Gamma \vdash A$  by assumption can be dropped: it is subsumed by the alternative using Lemma 4.1.2. However, the proof obtained can be different. Explain the difference and why the outright proof by assumption is subsumed.
- c) How would the [Haskell code](#) from the last tutorial need to be adapted to account for these improvements?



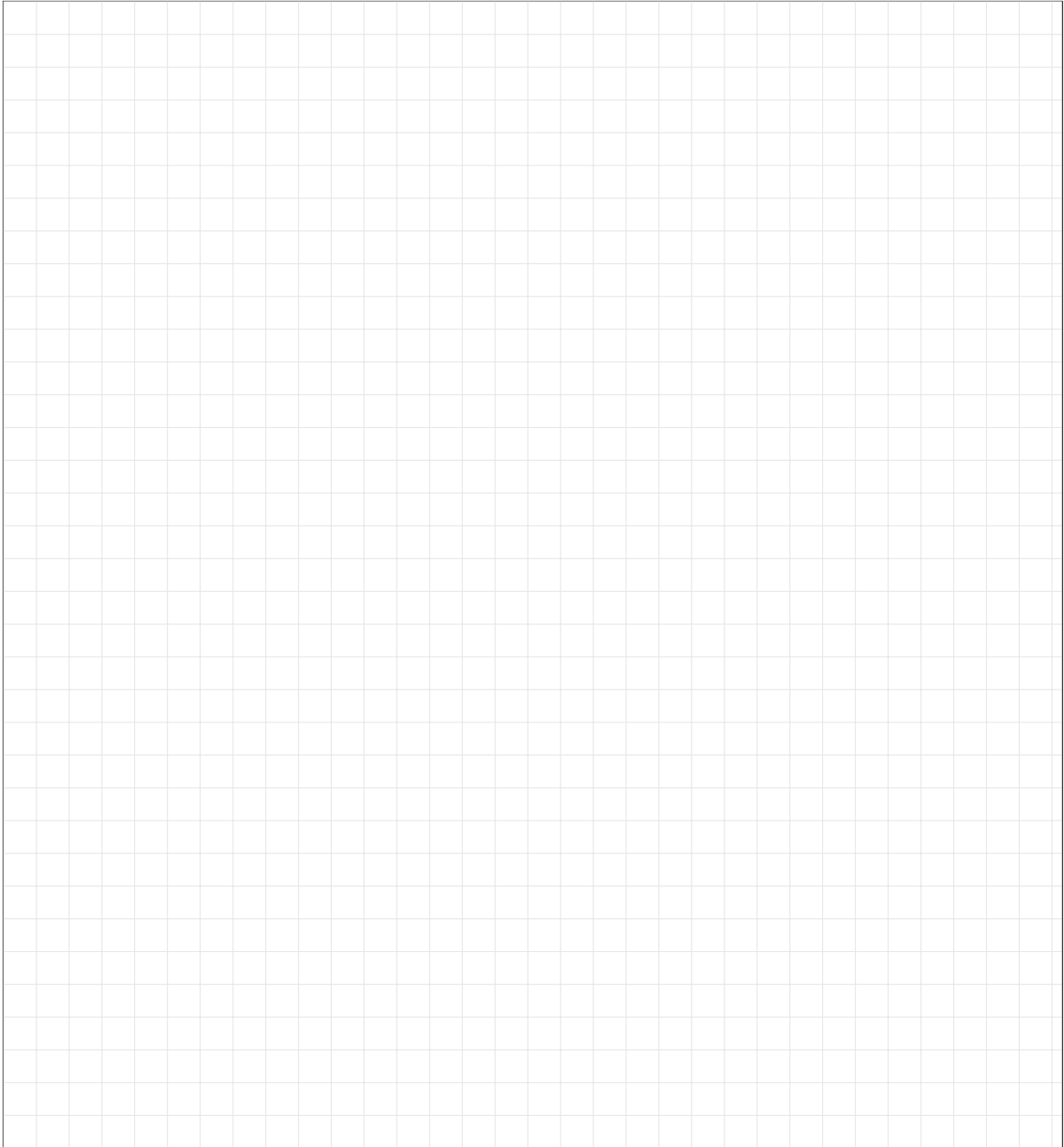
## Exercise 2 (Intuitionistic Proofs)

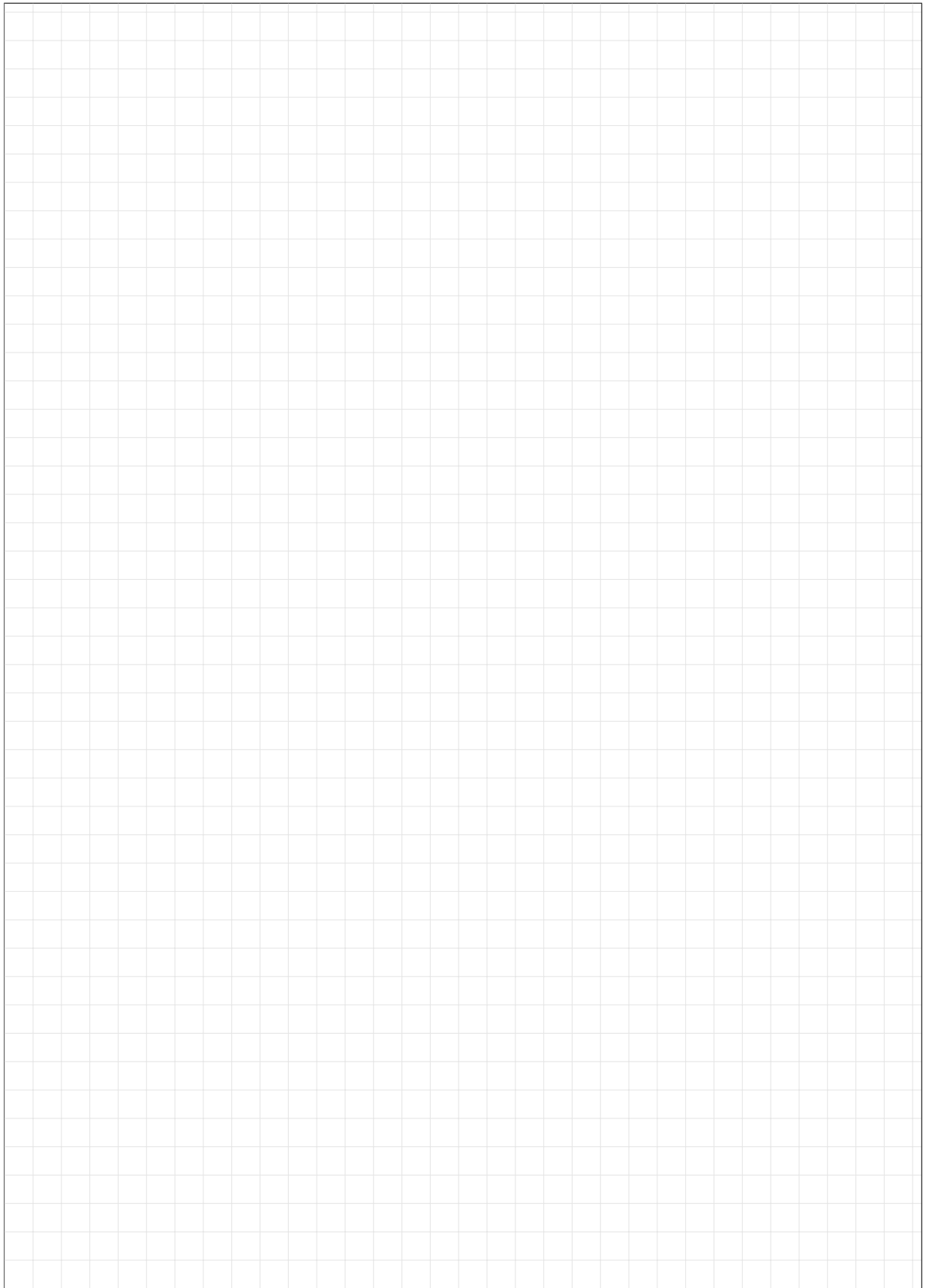
Prove the following propositions in intuitionistic logic:

a)  $(A \rightarrow A) \vee B$

b)  $A \rightarrow (B \rightarrow A \wedge B)$

c)  $(A \rightarrow C) \rightarrow ((B \rightarrow C) \rightarrow (A \vee B \rightarrow C))$





### Homework 3 (Weak Normalization with Pairs)

We previously proved (Exercise 8.2) that every type-correct  $\lambda^{\rightarrow}$ -term has a  $\beta$ -normal form. Adapt the proof to accommodate for the extension of the simply typed lambda calculus with pairs.

### Homework 4 (From Proof Terms to Propositions)

Consider the following proof term:

$$\lambda q. \lambda p. \text{case } \pi_1 p \text{ of } \text{inl } a \Rightarrow \text{inl } \langle \pi_1 q, \langle a, \pi_2 p \rangle \rangle \mid \text{inr } b \Rightarrow \text{inr } \langle \pi_2 q, b \rangle$$

- a) Exhibit the proposition that is proved by this term.
- b) Give the corresponding proof tree.

### Homework 5 (Intuitionistic Proofs)

Prove the following propositions in pure logic, without lambda-terms, and write down the  $\lambda$ -term corresponding to each proof:

- a)  $\neg(A \vee B) \rightarrow \neg A \wedge \neg B$
- b)  $\neg A \wedge \neg B \rightarrow \neg(A \vee B)$