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Lambda Calculus
Winter Term 2023/24
Exercise Sheet 12

Exercise 1 (Intuitionistic Proof Search)

The algorithm in Theorem 4.1.4 can be streamlined as follows:

a) When trying to prove $\Gamma \vdash A \to B$, it suffices to try ($\to$Intro). Explain why.

b) The attempt to prove $\Gamma \vdash A$ by assumption can be dropped: it is subsumed by the alternative using Lemma 4.1.2. However, the proof obtained can be different. Explain the difference and why the outright proof by assumption is subsumed.

c) How would the Haskell code from the last tutorial need to be adapted to account for these improvements?

Exercise 2 (Intuitionistic Proofs)

Prove the following propositions in intuitionistic logic:

a) $(A \to A) \lor B$

b) $A \to (B \to A \land B)$

c) $(A \to C) \to ((B \to C) \to (A \lor B \to C))$

Homework 3 (Weak Normalization with Pairs)

We previously proved (Exercise 8.2) that every type-correct $\lambda^{\rightarrow}$-term has a β-normal form. Adapt the proof to accomodate for the extension of the simply typed lambda calculus with pairs.

Homework 4 (From Proof Terms to Propositions)

Consider the following proof term:

$$\lambda q.\ \lambda p.\ \mathsf{case}\ \pi_1\ p\ \mathsf{of}\ \mathsf{inl}\ a \Rightarrow \mathsf{inl}\ {<}\pi_1\ q,\ {<}a,\ \pi_2\ p{>}{>}\ |\ \mathsf{inr}\ b \Rightarrow \mathsf{inr}\ {<}\pi_2\ q,\ b{>}$$

a) Exhibit the proposition that is proved by this term.

b) Give the corresponding proof tree.

Homework 5 (Intuitionistic Proofs)

Prove the following propositions in pure logic, without lambda-terms, and write down the λ-term corresponding to each proof:

a) $\neg(A \vee B) \rightarrow \neg A \wedge \neg B$

b) $\neg A \wedge \neg B \rightarrow \neg(A \vee B)$