

LOGIC EXERCISES

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EXERCISE SHEET 13

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The tutorial takes place on 13.07, 12–14.

Exercise 13.1. [Ferrante–Rackoff Elimination]

Apply the Ferrante–Rackoff Elimination to check the following sentence:

$$\exists x(\exists y(x = 2 \cdot y) \rightarrow (2 \cdot x \geq 0 \vee 3 \cdot x < 2))$$

Exercise 13.2. [Presburger Arithmetic]

Eliminate the quantifiers from the following formulas according to Presburger arithmetic:

1. $\forall y(3 < x + 2y \vee 2x + y < 3)$
2. $\forall x(2 \mid x \rightarrow (2x \geq 0 \vee 3x < 2))$

Exercise 13.3. [Quantifier Elimination for $Th(\mathbb{N}, 0, S, =)$]

Give a quantifier-elimination procedure for $Th(\mathbb{N}, 0, S, =)$ where S is the successor operation on natural numbers, i.e. $S(n) = n + 1$.

Hint: $a = b$ iff $S^k(a) = S^k(b)$ for any $a, b, k \in \mathbb{N}$.

Homework 13.1. [Under Presburger] (++)

Perform Presburger arithmetic quantifier elimination for each of the following formulas:

1. $\forall x \forall y (0 < y \wedge x < y \rightarrow x + 1 < 2y)$
2. $\forall x (\exists y (x = 2y \wedge 2 \mid y) \rightarrow 4 \mid x)$

Homework 13.2. [Quantifier Elimination for $Th(\mathbb{Z}, 0, S, P, =, <)$] (+++)

Give a quantifier-elimination procedure for $Th(\mathbb{Z}, 0, S, P, =, <)$ where S is the successor and P the predecessor operation on integers, i.e. $S(n) = n + 1$ and $P(n) = n - 1$. Do not use Presburger arithmetic; give a direct algorithm.

It is always easy to be logical.

It is almost impossible to be logical to the bitter end.

— Albert Camus