

Homework is due on May 11, before the tutorial.

Exercise 1 (H) (*Composition of Substitutions*)

- a) Specify $\text{Dom}(\sigma\tau)$ and $\sigma\tau$ for each of the following pairs of substitutions. Use the form $\{x \mapsto \cdot, \dots\}$ for the latter one.
- i) $\sigma = \{x \mapsto y\}, \tau = \{y \mapsto x\}$
 - ii) $\sigma = \{x \mapsto y * x\}, \tau = \{x \mapsto y\}$
 - iii) $\sigma = \{x \mapsto y\}, \tau = \{x \mapsto x * y\}$
- b) Specify conditions on the domains and variable ranges of σ and τ under which $\sigma\tau = \tau$. Show that your conditions are both necessary and sufficient.
- c) Show that we have $\sigma\tau = \sigma$ if $\forall x \in \text{Dom}(\sigma). \sigma(x) = \tau(x)$ and $\text{Dom}(\sigma) \cap \mathcal{VRan}(\tau) = \emptyset$. Is the converse also true in general?

Exercise 2 (H) (*Groups*)

Let \circ be a binary, \cdot^{-1} a unary, and e a nullary function symbol. Moreover, assume the following identities:

$$G = \{x \circ (y \circ z) \approx (x \circ y) \circ z, x \circ e \approx x, x \circ x^{-1} \approx e\}$$

Show $e \circ x \xrightarrow{*}_G x$.

Exercise 3 (T) (*Instances*)

Let $T(\Sigma, \{x\})$ be the set of Σ -terms over the single variable x . Moreover let the reduction relation \longrightarrow_I on $T(\Sigma, \{x\})$ be defined as follows: $s \longrightarrow_I t$ iff s is an instance of t , and $s \neq t$.

- a) Show that \longrightarrow_I is terminating and confluent.
- b) Does this result generalize to reduction systems with more than one variable?

Exercise 4 (T) (*Positions*)

Let s, t be terms, and Let $p, q \in \text{Pos}(s)$ be parallel positions in s (i.e., $p \parallel q$). Show $(s[t]_p)|_q = s|_q$.