

Homework is due on May 18, before the tutorial.

Exercise 1 (H) (*Terms*)

Prove the following lemmas by induction. Assume in each that $p \in \mathcal{Pos}(s)$ and $q \in \mathcal{Pos}(t)$.

- a) $(s[t]_p)|_{pq} = t|_q$
- b) $(s[t]_p)[r]_{pq} = s[t[r]_q]_p$

Exercise 2 (H) (*Equational Logic*)

Let \circ be a binary, \cdot^{-1} a unary, and e a nullary function symbol. Consider the following equation set G :

$$G = \{x \circ (y \circ z) \approx (x \circ y) \circ z, e \circ x \approx x, x^{-1} \circ x \approx e\}$$

Construct a *proof tree* for $G \vdash x \circ x^{-1} \approx ((x^{-1})^{-1} \circ x^{-1}) \circ (x \circ x^{-1})$. Make it clear what rule you use in each derivation step.

Exercise 3 (T) (*Models*)

Let $H = \{f(x, f(y, z)) \approx f(f(x, y), z), f(e, x) \approx x, f(x, i(x)) \approx e\}$.

- a) Construct a model of H where the additional equation $f(x, e) \approx x$ also holds.
- b) Show that $f(x, e) \approx_H x$ is not a semantic consequence of H , by constructing a model of H where $f(x, e) \approx x$ does *not* hold.

Exercise 4 (T) (*Equality*)

Let $\Sigma = \{f\}$ where f is a binary function symbol, and

$$E = \{f(x, f(y, z)) \approx f(f(x, y), z), f(x, x) \approx x, f(f(x, y), x) \approx f(x, y)\}$$

Is the question of whether two terms s and t are equivalent with respect to E decidable?
Hint: Figure out how to calculate, for any term s , the smallest term equivalent to s .