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Equational Logic
Summer Term 2012
Exercise Sheet 12
July 13

Homework is due on July 20th, before the tutorial.

Exercise 1 (H) (*Type Inference*)

Give a derivation tree for the following statement, and so determine the type τ:

$$[] \vdash \lambda xyz.\ x\ y\ (y\ z)\ : \tau$$

Exercise 2 (H) (*Subject Reduction Property*)

A type system has the *subject reduction property* if evaluating an expression preserves its type. Prove that the simply typed λ-calculus ($\lambda^{\to}$) has the subject reduction property:

$$\Gamma \vdash t : \tau \wedge t \longrightarrow_\beta t' \implies \Gamma \vdash t' : \tau$$

Hints: Use induction over the inductive definition of $\longrightarrow_\beta$ (Def. 1.2.2). State your inductive hypotheses precisely—it may help to introduce a binary predicate $P(t, t')$ to express the property you are proving by induction. Also note that the proof will require *rule inversion*: Given $\Gamma \vdash t : \tau$, the shape of t (variable, application, or λ-abstraction) may determine which typing rule must have been used to derive the typing judgment.

Within your proof, you are free to use the following lemma about substitution:

$$\Gamma \vdash u : \tau_0 \wedge \Gamma[x : \tau_0] \vdash t : \tau \implies \Gamma \vdash t[u/x] : \tau \tag{1}$$

Exercise 3 (T) (*Let-Polymorphism*)

Give a derivation tree for the following statement, and so determine the type τ:

$$[z : \tau_0] \vdash \text{ let } x = \lambda yz.\ z\ y\ y \text{ in } x\ (x\ z)\ : \tau$$

Exercise 4 (T) (*Recursive Let*)

Recursive let expressions are one way (besides Y-combinators) to add recursion to $\lambda^{\to}$.

$$t ::= x \mid (t_1\ t_2) \mid (\lambda x.\ t) \mid \text{letrec } x = t_1 \text{ in } t_2$$

a) Modify the standard typing rule for “let” to create a suitable rule for “letrec”.

b) Give a derivation tree for the following statement, and so determine the type τ:

$$[] \vdash \text{ letrec } x = \lambda y.\ x\ (x\ y) \text{ in } x\ x : \tau$$

Exercise 5 (T) (*Normal Form*)

Show that every type-correct $\lambda^{\to}$-term has a β-normal form.