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Equational Logic
Summer Term 2015
Exercise Sheet 4

Exercise 20 (Models)

a) Let $G = \{f(x, f(y, z)) \approx f(f(x, y), z)\}$.

   i) Construct a model of G.

   ii) Construct an f-algebra $\mathcal{A}$ such that $\mathcal{A}$ is not a model of G.

b) Let $H = \{f(x, f(y, z)) \approx f(f(x, y), z), f(e, x) \approx x, f(x, i(x)) \approx e\}$.

   i) Construct a model of H where the additional equation $f(x, e) \approx x$ also holds.

   ii) Show that $f(x, e) \approx_H x$ is not a semantic consequence of H, by constructing a model of H where $f(x, e) \approx x$ does *not* hold.

   iii) Why is this not a contradiction to Exercise 15 from last week's sheet?

Exercise 21 (Equality)

Let $\Sigma = \{f\}$ where f is a binary function symbol, and

$$E = \{f(x, f(y, z)) \approx f(f(x, y), z),\ f(x, x) \approx x,\ f(f(x, y), x) \approx f(x, y)\}$$

Is the question of whether two terms s and t are equivalent with respect to E decidable?

Hint: Figure out how to calculate, for any term s, the smallest term equivalent to s.

Homework 22 (Programming Assignment)

You are assigned to implement equational logic in Haskell based on the inference rules described in the book in §3.1 (p. 42). Additional instructions can be found on the lecture web page.

Homework 23 (Commutativity)

Let $\Sigma = \{0, s, +\}$ and $E = \{x + 0 \approx x,\ x + s(y) \approx s(x) + y\}$.

a) Show that $E \models x + s(s(0)) \approx s(s(x))$ and $E \models s^i(0) + s^j(0) \approx s^j(0) + s^i(0)$.

b) Show that $E \not\models x + y \approx y + x$.

   Hint: Give a model of E where $+$ is not commutative.