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Equational Logic
Summer Term 2015
Exercise Sheet 5

Exercise 24 (Equivalence Classes)

Let $\Sigma = \{f, g\}$ and

$$E = \{f(f(x)) \approx f(x),\ g(f(x)) \approx f(x),\ f(g(x)) \approx g(x),\ g(g(x)) \approx g(x)\}$$

a) Describe the equivalence classes of $\approx_E$.

b) For each equivalence class $[t]_{\approx_E}$, determine a shortest term $\widehat{t}$ in $[t]_{\approx_E}$.

c) Give a model for E that has more than one element.

Exercise 25 (Congruence Closure)

Let $\Sigma = \{f, a, b\}$ and $G = \{f(a, b) \approx a\}$. Using congruence closure, decide whether the terms s and t are equivalent with respect to the equation set G. Use the abstract algorithm on equation sets.

a) $s = f(f(a, b), b), \quad t = a$

b) $s = f(f(a, b), b), \quad t = b$

Exercise 26 (Unification)

Let u, x, y, and z be variables. Use the unification algorithm recalled in the lecture to solve the following two unification problems:

a) $S_1 = \{f(h(x), g(x, u)) \stackrel{?}{=} f(z, g(f(y, y), z))\}$

b) $S_2 = \{h(x, g(x, y), y) \stackrel{?}{=} h(x, g(a, y), y), z \stackrel{?}{=} h(x, g(x, b), b)\}$

Homework 27 (Programming Assignment)

You are assigned to implement congruence closure in Haskell. Additional instructions can be found on the lecture web page.

Homework 28 (More General Substitutions)

Let σ and σ' be substitutions such that each one is an instance of the other: Formally, we have both $\sigma \lesssim \sigma'$ and $\sigma \gtrsim \sigma'$. Show that in this case, there must exist a renaming ρ (i.e., an injective substitution where $\mathcal{R}an(\rho) \subseteq V$) such that $\sigma = \rho\sigma'$.