Technische Universität München
Institut für Informatik
Prof. Tobias Nipkow, Ph.D.
Johannes Hölzl and Lars Hupel

Equational Logic
Summer Term 2015
Exercise Sheet 9

Exercise 45 (Linear Term Rewriting Systems)

A rewrite rule $l \longrightarrow r$ is called *left-linear* if every variable in l occurs exactly once. Similarly, $l \longrightarrow r$ is called *right-linear* if every variable in r occurs exactly once. A rule is *linear* if it is both right- and left-linear. We say that a term rewriting system is *linear* if it contains only linear rules.

Show:

a) Every linear term rewriting system R that has no critical pairs is confluent. Give a self-contained proof; do not simply apply Corollary 6.3.11 from the book!

   Hint: Show that R is strongly confluent.

b) If R is a linear term rewriting system, and for every critical pair (t_1, t_2) there exists t_0 such that $t_1 \overset{=}{\longrightarrow} t_0 \overset{=}{\longleftarrow} t_2$, then R is confluent.

   Hint: Extend the proof of the previous statement.

Exercise 46 (λ-Terms)

Evaluate the following substitutions:

a) $(\lambda y.x(\lambda x.x))\ [(\lambda y.xy)/x]$

b) $(y(\lambda v.xv))\ [(\lambda y.vy)/x]$

Rewrite the following terms such that they are completely parenthesized and conform to the grammar for the λ-calculus given in the lecture (without any shortcut notations).

c) $ux(yz)(\lambda v.vy)$

d) $(\lambda xyz.xz(yz))uvw$

Rewrite the following terms such that there are as few parentheses as possible, and apply all shortcut notation from the lecture:

e) $((u(\lambda x.\ (v(wx))))x)$

f) $(((w(\lambda x.(\lambda y.(\lambda z.((xz)(yz))))))u)v)$

Exercise 47 (Formalizations with λ-Terms)

Express the following propositions as λ-terms. Use the constant D as a derivative operator.

a) The derivative of x^2 is $2x$.

b) The derivative of x^2 at 3 is 6.

c) Let f be a function, and let g be defined as $g(x) := f(x^2)$. The derivative of g at x is different from the derivative of f at x^2.

d) Formulate the proposition c) without using the auxiliary function symbol g.

Homework 48 (Strong Confluence)

Let $\rightsquigarrow$ be a relation with $\stackrel{=}{\rightarrow} \subseteq \rightsquigarrow \subseteq \stackrel{*}{\rightarrow}$.

Show that $\rightsquigarrow$ is strongly confluent iff $\forall t_1 t_2 s.\ t_1 \leftarrow s \rightsquigarrow t_2 \implies \exists t.\ t_1 \rightsquigarrow t \stackrel{*}{\leftarrow} t_2$.

(Strong confluence of $\rightarrow$ is $\forall t_1 t_2 s.\ t_1 \leftarrow s \rightarrow t_2 \implies \exists t.\ t_1 \stackrel{*}{\rightarrow} t \stackrel{=}{\leftarrow} t_2$)

Homework 49 (Confluence)

Let R be the following term rewriting system:

$$\{f(x,x) \longrightarrow a,\ c \longrightarrow g(c),\ g(x) \longrightarrow f(x, g(x))\}$$

Is R confluent? Justify your answer.

Homework 50 (Substitution Lemma)

Show that, given $x \neq y$ and $x \notin \mathrm{FV}(u)$:

$$s[t/x][u/y] = s[u/y][t[u/y]/x]$$