

# LOGICS EXERCISE

TU MÜNCHEN  
INSTITUT FÜR INFORMATIK

PROF. TOBIAS NIPKOW  
DR. PETER LAMMICH  
SIMON WIMMER

SS 2016

EXERCISE SHEET 4

04.05.2016

**Submission of Homework:** Before tutorial on May 11

## Exercise 4.1. [(In)finite Models]

1. Show that any model (for a formula of predicate logic) with an universe of size  $n$  can be extended to a model of size  $m$  for any  $m \geq n$ . Can it also be extended to an *infinite* model?
2. Now consider the extension of predicate logic with equality. Does above property still hold?

## Exercise 4.2. [Decidability and Context-Free Grammars]

Give an alternative proof that is impossible to decide validity of predicate logic formulas by using an encoding of context-free grammars in predicate logic.

*Hint:* Consider Chomsky normal forms. It is impossible to decide if two context-free languages are disjoint.

**Homework 4.1.** [Decidability of Consequence] (5 points)

Given a finite set  $M$  of (predicate logic) formulas, and a formula  $F$ . Is it semi-decidable whether  $M \models F$ ? Is it even decidable? Justify your answers!

**Homework 4.2.** [Ground Resolution] (5 points)

Use ground resolution to prove that the following formula is valid:

$$(\forall x P(x, f(x))) \longrightarrow \exists y P(c, y)$$

**Homework 4.3.** [Formulas without Negation] (5 points)

Prove that every predicate logic formula that only contains  $\wedge, \vee, \forall, \exists, \longrightarrow$  and atomic formulas is satisfiable. Is such a formula also valid?

**Homework 4.4.** [Herbrand Models] (5 points)

Given the formula

$$F = \forall x \forall y (P(f(x), g(y)) \wedge \neg P(g(x), f(y)))$$

- Specify a Herbrand model for  $F$ .
- Specify a Herbrand structure suitable for  $F$ , which is not a model of  $F$ .