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SS 2016 | Exercise Sheet 11 | 22.06.2016

Submission of Homework: Before tutorial on June 29

Exercise 11.1. [Natural Deduction (Warmup)]
Prove by natural deduction:

1. $(F \wedge G) \wedge H \to F \wedge (G \wedge H)$
2. $(F \vee G) \vee H \to F \vee (G \vee H)$

Exercise 11.2. [Natural Deduction (Advanced)]
Prove by natural deduction:

1. $\neg(F \wedge G) \to (\neg F \vee \neg G)$
2. $((F \to G) \to F) \to F$

Exercise 11.3. [Alternative $\wedge$E rule]
Show how to transform a natural deduction proof that additionally uses the following rule to one that does not use the rule:

$$\frac{F \wedge G \qquad \begin{array}{c}[F, G]\\ \vdots \\ H\end{array}}{H}$$

Homework 11.1. [Natural Deduction (Warmup)] (3 points)
Show:

$$\vdash_N (F \to G) \to (\neg G \to \neg F)$$

Homework 11.2. [Classical Reasoning (1)] (6 points)
We replace rule $\bot$ of the calculus of natural deduction by either one of the following rules:

- $\dfrac{}{F \vee \neg F}$ (law of excluded middle)
- $\dfrac{\neg\neg F}{F}$ (double negation elimination)

Additionally, we add the rule $\dfrac{\bot}{F}$ $(\bot E)$. Show that the calculus of natural deduction remains complete in both cases.

Homework 11.3. [Classical Reasoning (2)] (5 points)
Assume that the calculus of natural deduction is *augmented* with the two rules from the last exercise. Show:

- $\vdash_N (\neg G \to \neg F) \to (F \to G)$
- $\vdash_N (\neg F \to G) \to (F \vee G)$

Hint: You can also use the law of excluded middle in the following form:

$$\dfrac{\begin{matrix}[F] \\ \vdots \\ G\end{matrix} \qquad \begin{matrix}[\neg F] \\ \vdots \\ G\end{matrix}}{G}$$

Homework 11.4. [Left-Sided Sequent Calculus] (6 points)
We want to study a modified sequent calculus where the right-hand side is always empty, i.e. where sequents are of the form $\Gamma \Rightarrow$. Give a set of rules for this calculus such that your calculus fulfills the following property and sketch a proof:

$$\Gamma \Rightarrow \Delta \text{ iff } \Gamma, \neg\Delta \Rightarrow$$

Hint: Use induction over the length of the derivations. You can skip the cases for $\vee$ and $\wedge$ and instead look at $\to$.