

# LOGICS EXERCISE

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EXERCISE SHEET 4

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**Submission of homework:** Before tutorial on 30.05.2017. You have to do the homework yourself; no teamwork allowed.

**Exercise 4.1. [Natural Deduction]**

Prove the following formulas by natural deduction:

1.  $(F \wedge G) \wedge H \rightarrow F \wedge (G \wedge H)$
2.  $(F \vee G) \vee H \rightarrow F \vee (G \vee H)$
3.  $\neg(F \wedge G) \rightarrow (\neg F \vee \neg G)$

**Exercise 4.2. [Induction Proofs]**

Show that the following schema has a proof in natural deduction for all  $n \geq 1$ :

$$P_n = ((A_1 \wedge (A_2 \wedge (\dots \wedge A_n) \dots)) \rightarrow B) \rightarrow (A_1 \rightarrow (A_2 \rightarrow (\dots (A_n \rightarrow B) \dots)))$$

**Exercise 4.3. [Classical Reasoning]**

We replace rule  $\perp$  of the calculus of natural deduction by either one of the following rules:

- $\frac{}{F \vee \neg F}$  (law of excluded middle)
- $\frac{\neg\neg F}{F}$  (double negation elimination)

Additionally, we add the rule  $\frac{\perp}{F}$  ( $\perp E$ ). Show that the calculus of natural deduction remains complete in both cases.

**Homework 4.1.** [Natural Deduction]

(10 points)

Prove the following formulas by natural deduction (as specified in the lecture):

1.  $((A \rightarrow B) \rightarrow A) \rightarrow A$
2.  $(A \vee B) \rightarrow \neg(\neg A \wedge \neg B)$
3.  $(\neg G \rightarrow F) \rightarrow (\neg F \rightarrow G)$

**Homework 4.2.** [Substitution]

(10 points)

Assume that there are proofs for  $\vdash_N G \rightarrow G'$  and  $\vdash_N G' \rightarrow G$ . Construct the proof for  $\vdash_N F[G/A] \rightarrow F[G'/A]$ .