

LOGICS EXERCISE

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EXERCISE SHEET 12

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Submission of homework: Before tutorial on 25.07.2017. You have to do the homework yourself; no teamwork allowed.

Exercise 12.1. [Fourier–Motzkin Elimination]

1. $\exists x \exists y (2 \cdot x + 3 \cdot y = 7 \wedge x < y \wedge 0 < x)$

2. $\exists x \exists y (3 \cdot x + 3 \cdot y < 8 \wedge 8 < 3 \cdot x + 2 \cdot y)$

Exercise 12.2. [Ferrante–Rackoff Elimination]

$$\exists x (\exists y (x = 2 \cdot y) \rightarrow (2 \cdot x \geq 0 \vee 3 \cdot x < 2))$$

Exercise 12.3. [Presburger Arithmetic]

1. $\forall y (3 < x + 2 \cdot y \vee 2 \cdot x + y < 3)$

2. $\forall x (\exists y (x = 2 \cdot y) \rightarrow (2 \cdot x \geq 0 \vee 3 \cdot x < 2))$

Exercise 12.4. [Quantifier Elimination for $Th(\mathbb{N}, 0, S, =)$]

Give a quantifier-elimination procedure for $Th(\mathbb{N}, 0, S, =)$ where S is the successor operation on natural numbers, i.e. $S(n) = n + 1$.

Hint: $a = b$ iff $S^k(a) = S^k(b)$ for any $a, b, k \in \mathbb{N}$.

Homework 12.1. [Quantifier Elimination] (6 points)

Perform Presburger arithmetic quantifier elimination (Cooper's algorithm) for each formula:

1. $\forall x \forall y (0 < y \wedge x < y \rightarrow x + 1 < 2 \cdot y)$
2. $\forall x (\exists y (x = 2 \cdot y \wedge 2 \mid y) \rightarrow 4 \mid x)$

Homework 12.2. [Garnix Problem] (6 points)

A university brewery has two kegging machines (M_1 and M_2) and produces two products, beer (B) and shandy (R). M_1 needs 50 minutes to fill a keg of beer but only 24 minutes to fill a keg of shandy, while M_2 needs 30 minutes for a keg of either one drink.

For a student festival beginning two weeks from now, an order of 75 kegs of beer and 95 kegs of shandy has been placed, which has to be fulfilled during the upcoming week. However, machine M_1 has to undergo maintenance and therefore is expected to be running between 37 and 42 hours, while M_2 can be operated up to 100 hours during the whole week (and as few hours as desirable).

Use Fourier–Motzkin and Ferrante–Rackoff elimination to check whether the brewery can hold up to its promise.

Homework 12.3. [Quantifier Elimination for $Th(\mathbb{N}, 0, S, =, <)$] (8 points)

Similarly to Exercise 12.4, give a quantifier-elimination procedure for $Th(\mathbb{N}, 0, S, =, <)$.