Logics Exercise		
TU München Institut für Informatik		Prof. Tobias Nipkow Lars Hupel
SS 2018	Exercise Sheet 4	30.04.2018

Submission of homework: Before tutorial on 08.05.2018. Until further notice, homework has to be submitted in groups of two students.

Exercise 4.1. [Atomic Cut]
Let A be an atomic formula. Prove that if $\vdash_G \Gamma \Rightarrow A, \Delta$ and $\vdash_G A, \Gamma \Rightarrow \Delta$, then $\vdash_G \Gamma \Rightarrow \Delta$.

Exercise 4.2. [Natural Deduction]
Prove the following formulas by natural deduction:

1. $(F \wedge G) \wedge H \rightarrow F \wedge (G \wedge H)$
2. $(F \vee G) \vee H \rightarrow F \vee (G \vee H)$
3. $\neg(F \wedge G) \rightarrow (\neg F \vee \neg G)$

Exercise 4.3. [Classical Reasoning]
We replace rule $\bot$ of the calculus of natural deduction by either one of the following rules:

- $\dfrac{}{F \vee \neg F}$ (law of excluded middle)
- $\dfrac{\neg\neg F}{F}$ (double negation elimination)

Additionally, we add the rule $\dfrac{\bot}{F}$ ($\bot E$). Show that the calculus of natural deduction remains complete in both cases.

Homework 4.1. **[Natural Deduction]** (10 points)
Prove the following formulas by natural deduction (as specified in the lecture):

1. $((A \to B) \to A) \to A$
2. $(\neg G \to F) \to (\neg F \to G)$
3. $\neg\neg\neg F \to \neg F$ (without using the $\bot$ rule, but the $\bot E$ rule from Exercise 4.3 is allowed)

Homework 4.2. **[Substitution]** (10 points)
Assume that there are proofs for $\vdash_N G \to G'$ and $\vdash_N G' \to G$. Construct the proof for $\vdash_N F[G/A] \to F[G'/A]$.