Logics Exercise

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SS 2018 — Exercise Sheet 9 — 05.06.2018

Submission of homework: Before tutorial on 12.06.2018. Until further notice, homework has to be submitted in groups of two students.

Exercise 9.1. **[Decidability]**

1. Resolution for first-order logic is sound and complete.
2. Satisfiability and validity for first-order logic are undecidable.

How do you reconcile these two facts?

Exercise 9.2. **[Ground Resolution]**
Use ground resolution to prove that the following formula is valid:

$$(\forall x P(x, f(x))) \longrightarrow \exists y P(c, y)$$

Exercise 9.3. **[Equality]**
We consider how to model equality in predicate logic. In the lecture slides, the following axiom schema for congruence is used:

$$\frac{Eq(x_i, y)}{Eq(f(x_1, \ldots, x_i, \ldots, x_n), f(x_1, \ldots, y, \ldots, x_n))}$$

Assume that this schema is replaced by:

$$\frac{Eq(x_1, y_1) \qquad \cdots \qquad Eq(x_n, y_n)}{Eq(f(x_1, \ldots, x_n), f(y_1, \ldots, y_n))}$$

Reflexivity, symmetry and transitivity stay unchanged. Show that the above modified schemas is equivalent to the schemas from the lecture.

Hint: Simulate the modified schema with the original one and vice versa.

Homework 9.1. **[Restricted Resolution]** (8 points)
In the resolution procedure as defined in the lecture slides, we can unify arbitrarily many literals from two clauses. Consider a modified resolution procedure, where exactly one literal is picked. We add another rule ("collapsing rule"): For a clause $C = \{L_1, \ldots, L_n\}$, where $\{L_i, L_j\}$ can be unified using a mgu δ, add another clause $C' = (C - L_i)\delta$.

For example, given the clause

$$C = \{\neg W(x), \neg W(f(y)), T(x, y), \neg W(f(c))\}$$

we can apply the collapsing rule as follows:

$$L_1 = \neg W(x), L_2 = \neg W(f(y)), \delta = \{x \mapsto f(y)\}, C' = \{\neg W(f(y)), T(f(y), y), \neg W(f(c))\}$$

(Note that there are multiple possible ways to apply the collapsing rule to C.)

Prove that our modified resolution calculus, including collapsing rule, can be simulated by the original resolution calculus, and vice versa.

Homework 9.2. **[Resolution]** (6 points)
Show with resolution that:

$$f(f(f(a))) = a \longrightarrow f(f(a)) = a \longrightarrow f(a) = a$$

is valid. First, remove equality based on the procedure from the lecture. Then perform resolution.