

Semantics of Programming Languages

Exercise Sheet 13

For a change, this exercise should be solved on paper only, not using Isabelle.

Exercise 13.1 Procedure Definedness Check

We consider a language with statically scoped procedures but (for simplicity) without local variables. For this language we can define a small-step semantics that does not require a special procedure environment. Instead, the context of procedure declarations is managed by gradually transforming the program itself. The rules for the basic commands remain unchanged:

$$(x ::= a, s) \rightarrow (SKIP, s(x := \text{aval } a \ s))$$

$$(SKIP; c_2, s) \rightarrow (c_2, s)$$

$$(c_1, s) \rightarrow (c_1', s') \implies (c_1; c_2, s) \rightarrow (c_1'; c_2, s')$$

$$\text{bval } b \ s \implies (IF \ b \ THEN \ c_1 \ ELSE \ c_2, s) \rightarrow (c_1, s)$$

$$\neg \text{bval } b \ s \implies (IF \ b \ THEN \ c_1 \ ELSE \ c_2, s) \rightarrow (c_2, s)$$

$$(WHILE \ b \ DO \ c, s) \rightarrow (IF \ b \ THEN \ c; \ WHILE \ b \ DO \ c \ ELSE \ SKIP, s)$$

Now, procedure declarations distribute over semicolons, and disappear when they surround a *SKIP*. Moreover, we may make an arbitrary step under a procedure declaration:

$$(\{PROC \ p = cp;; \ c_1; \ c_2\}, s) \rightarrow (\{PROC \ p = cp;; \ c_1\}; \{PROC \ p = cp;; \ c_2\}, s)$$

$$(\{PROC \ p = cp;; \ SKIP\}, s) \rightarrow (SKIP, s)$$

$$(c, s) \rightarrow (c', t) \implies (\{PROC \ p = cp;; \ c\}, s) \rightarrow (\{PROC \ p = cp;; \ c'\}, t)$$

- (a) Complete the small-step semantics by formulating the missing rules for *CALL*.
- (b) Define a recursive function that checks if a program is well-formed, that is, it contains no calls to procedures that were not defined.
- (c) Prove that the evaluation of a well-formed program cannot get stuck: If c is well-formed and $(c, s) \rightarrow^* (c', t)$ and $\text{final } (c', t)$ then $c' = SKIP$.

Recall that $\text{final } cs$ is defined as $\neg (\exists cs'. cs \rightarrow cs')$.